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Explicit Breaking of Supersymmetry by Non-Perturbative Effects

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ABSTRACT

Instanton effects in a family of completely massive Higgs models with N=1 supersymmetry are investigated. The models have $N_c = 2$ and $N_f \geq 2$. In each model, we show that a certain gauge invariant correlation function depends in a non-trivial way on its coordinates, in spite of the fact that supersymmetry requires its constancy. This means that non-perturbative effects break supersymmetry explicitly in the one instanton sector. We also show that condensates arising in the point-like limit of the above correlation functions can in principle be used to induce the Electro-Weak scale.

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1. Introduction

Non-perturbative effects in asymptotically free, supersymmetric gauge theories have been investigated extensively for more than a decade [1-10]. Recently there has been a renewed interest in the subject [11].

Crucial to almost the entire literature on the subject, is the *assumption* that non-perturbative effects do not break supersymmetry (SUSY) explicitly. This applies in particular to the study of dynamical (spontaneous) SUSY breaking [1-5]. There have been attempts to verify the validity of that assumption [4-8], but with no conclusive results.

In a continuum framework, the leading non-perturbative effect arises from the one instanton sector. For a quantity that vanishes to all orders in perturbation theory, the one instanton result represents the leading order contribution in a systematic expansion *provided* the gauge coupling is *weak*. For this reason, we restrict our attention in this paper to SUSY-Higgs models.

We calculate various gauge invariant correlation functions in a family of SU(2)-Higgs models with N=1 SUSY. The computation involves little more than semi-classical instanton calculus, and it reveals the existence of explicit SUSY breaking effects in the one instanton sector.

In each model, the classical potential has a unique supersymmetric minimum. The Higgs VEV breaks the SU(2) gauge symmetry completely, and all fields acquire non-zero masses at tree level. Because of the absence of massless fermions there is no room for spontaneous SUSY breaking, as there is no candidate to become a goldstino[1]. We show that, nevertheless, a certain gauge invariant correlation function violates a SUSY Ward identity. This proves that SUSY is broken explicitly in the one instanton sector. The Ward identity has been chosen to minimize the amount of technicalities involved in the computation. The simplest model has $N_f = 2$ in the terminology of ref. [3], and in App. B we generalize our results to $N_f > 2$.

In more detail, we first show that a certain bosonic condensate is formed. The operator which condenses takes the form $\Gamma(x,x)$, where $\Gamma(x,y)$ is the correlator of two gauge invariant composite operators. We then show that $\Gamma(x,y) \to 0$ as $|x-y| \to \infty$. The discrepancy with SUSY arises because a SUSY Ward identity requires the correlator $\Gamma(x,y)$ to be independent of the separation $x-y$. We expect that the computation of other quantities of physical interest, such as non-perturbative corrections to boson and fermion masses, will reveal further violations of SUSY.

[1] Since the broken gauge coupling can be taken to be as weak as we like, we can safely assume that there are no composite massless states.

This paper is organized as follows. In Sect. 2 we define the basic SUSY-Higgs model. In Sect. 3 we discuss the one instanton sector and find that a certain bosonic condensate is formed. In Sect. 4 we show that SUSY is explicitly broken in the one instanton sector. In Sect. 5 we make a first excursion into the phenomenological implications of our result. We show that condensates of the kind described above can in principle be used to induce the Electro-Weak scale. Sect. 6 contains a short discussion. In App. A we discuss the asymptotic behaviour of the zero modes. Finally, in App. B we show that violations of SUSY occur in models with arbitrary $N_f \geq 2$.

2. The model

The model we present in this section is a variant of one of the simplest SUSY Higgs models. This model serves as a test case. We show that a certain SUSY Ward identity pertaining to a gauge invariant correlation function is violated explicitly. Our results also have phenomenological implications. We find that certain bosonic condensates are formed quite generally in the one instanton sector. If these condensates arise from GUT scale physics or, alternatively, from a strongly interacting hidden sector at the TeV range, they can in principle be used to induce the Electro-Weak scale in the observed sector.

The basic ingredient of the model is an SU(2)-Higgs sector. It consists of a gauge supermultiplet (A^a_μ, λ^a) and of several chiral supermultiplets. $\Phi_{iA} = (\phi_{iA}, \psi_{iA})$ contain the Higgs fields and their fermionic partners. In addition, there is a neutral supermultiplet $\Phi^0 = (\phi^0, \psi^0)$. The higgs fields are doublets of both $\mathrm{SU}(2)_c$ and $\mathrm{SU}(2)_f$, and $A, i = 1, 2$ are respectively the colour and flavour indices. We let T^a and F^a denote the colour and flavour generators respectively. In this paper, we use the representation $T^a_{AB} = -\frac{1}{2}\sigma^a_{BA}$ and $F^a_{ij} = \frac{1}{2}\sigma^a_{ij}$.

The superpotential is

$$W_1 = h\Phi^0 \left(\frac{1}{2} \epsilon_{ij}\epsilon_{AB}\Phi_{iA}\Phi_{jB} - v^2 \right) . \tag{1}$$

The classical potential has a unique (up to colour and flavour transformations) SUSY minimum, which can be chosen to be

$$\langle \phi_{iA} \rangle = v\delta_{iA} \, . \tag{2}$$

The minimum breaks the gauge symmetry completely, and it leaves unbroken the diagonal $\mathrm{SU}(2)_V$ generated by $T^a + F^a$. Under $\mathrm{SU}(2)_V$, the two Higgs superfields decompose into a singlet $\Phi' = \delta_{iA}\Phi_{iA}/\sqrt{2}$ and a triplet $\Phi^a = \sigma^a_{iA}\Phi_{iA}/\sqrt{2}$.

All fields acquire non-zero masses at tree level. The massive gauge supermultiplet has mass $\mu = gv$ where g is the gauge coupling. Its bosonic and fermionic components are respectively A^a_μ and $\mathrm{Re}\,\phi^a$, and λ^a and $\bar\psi^a$. (The fields $(\lambda^a, \bar\psi^a)$ form a massive Dirac spinor). In the singlet sector, the mass is $m = \sqrt{2}hv$, and the component fields are ϕ^0, ϕ', ψ^0 and $\bar\psi'$.

We add to the model two "lepton" families. The corresponding chiral superfields are $\eta^\pm_A$ and $\xi^\pm_i$. These letters will also be used to denote the fermionic components, whereas a tilde over the letter is used to denote the scalar components. The $\pm$ superscript actually correspond to a new SU(2) "family" symmetry. Apart from the fact that there are two η-s and two ξ-s, the family SU(2) will play little role below.

The full superpotential is $W = W_1 + W_2$, where

$$W_2 = y\,\epsilon_{ij}\epsilon_{AB}\,\Phi_{jB}\left(\xi^+_i\,\eta^-_A - \xi^-_i\,\eta^+_A\right)\,. \tag{3}$$

The two "lepton" families are massive too, and their mass is $m_1 = yv$. The Dirac spinors are $(\eta^\pm, \bar\xi^\mp)$. A summary of the field content of the model can be found in Table 1. This table also gives the charges of the fermions under the non-anomalous R-symmetry. For chiral superfields, the charges of the corresponding scalars are related by $Q_R(scalar) = Q_R(fermion) + 1$.

3. The one instanton sector

There are standard techniques to compute correlation functions in the one instanton sector of any Higgs model. One integrates over a family of classical backgrounds labeled by collective coordinates, and for every background one has a systematic expansion in powers of the coupling constant(s). In a SUSY-Higgs model there are exact fermionic zero modes in spite of the fact that some (or all) fields are massive. This feature, however, is not unique to SUSY theories, and it is present already in Electro-Weak sector of the Standard Model. The physical significance of the fermionic zero modes and the techniques for dealing with them have been discussed by 'tHooft [12].

In this paper we follow the conventions of ref. [3] with minor modifications. The classical gauge field is given by

$$A^a_\mu = \frac{2}{g}\,a(r)\,\bar\eta_{a\mu\nu}\,(x^\mu - x^\mu_0)\,, \tag{4}$$

where $r^2 = (x - x_0)^2$. The collective coordinates x^μ_0 describe the instanton's center. The function $a(r)$ tends to a non-zero constant at the origin, and its asymptotic behaviour is $a(r) \sim 1/r^2$. In the case of an unbroken gauge symmetry, one has

$a(r) = 1/(r^2 + \rho^2)$ where ρ is the instanton's size. In the Higgs case, the constrained instanton [13] is still characterized by a scale parameter ρ, but the precise form of $a(r)$ is different.

The Higgs field has the following form

$$\phi_{iA} = iv\,\bar{\sigma}^{\mu}_{iA}\,(x^{\mu} - x^{\mu}_{0})\,\varphi(r)\,. \tag{5}$$

The real function $\varphi(r)$ tends to a constant at small r, whereas its asymptotic behaviour is $\varphi(r) \sim 1/r$. Finally, the conserved angular momenta in the instanton background are

$$\begin{aligned} K^a_1 &= S^a_1 + L^a_1 + T^a\,, \\ K^a_2 &= S^a_2 + L^a_2 + F^a\,. \end{aligned} \tag{6}$$

We now turn to the fermionic zero modes. In the absence of a Higgs VEV, the model had had four gaugino zero modes and four matter zero modes (one for each charged doublet). With the Higgs VEV, four of the zero modes disappear [3]. Two zero modes survive in the SU(2)-Higgs sector. We refer to them as the "gaugino" zero modes. In addition, every "lepton" family contributes one zero mode. Another feature is that, in the Higgs case, each zero mode contains more than one channel. The "gaugino" zero modes have the following decomposition (for $x^{\mu}_0 = 0$)

$$\begin{aligned} (\lambda^a_{\alpha})_k &= \sigma^a_{\alpha k}\,f(r)\,, \\ (\psi^{\dagger}_{iA\alpha})_k &= i\delta_{i\alpha}\delta_{Ak}\,g(r) + ix^{\mu}x^{\nu}\sigma^{\mu}_{Ai}\bar{\sigma}^{\nu}_{\alpha k}\,h(r)\,, \\ (\psi^0_{\alpha})_k &= i\delta_{\alpha k}\,p(r)\,. \end{aligned} \tag{7}$$

Here α is the spinor index, and the index $k = 1, 2$ counts the two zero modes. We use the notation $\psi^{\dagger}_{\alpha} = \epsilon_{\alpha\beta}\bar{\psi}_{\beta}$. The "lepton" zero modes are

$$\begin{aligned} \eta^{\pm}_{\alpha A} &= \delta_{\alpha A}\,u(r)\,, \\ \xi^{\dagger\mp}_{\alpha i} &= \mp\delta_{\alpha i}\,v(r)\,. \end{aligned} \tag{8}$$

The quantum numbers of the four zero modes as well as their different channels can be found in Table 2.

For each zero mode, the radial functions solve a set of ordinary coupled differential equations. These equations can be found in App. A, which also gives the asymptotic large-r behaviour of the zero modes. The small-r behaviour will not be needed below.

In the rest of this section we will show that a certain bosonic condensate is formed in the one instanton sector. In the next section we show that, with slight modification,

this condensate can be regarded as the point-like limit of a gauge invariant correlation function, and that that correlation function violates SUSY by failing to be a constant.

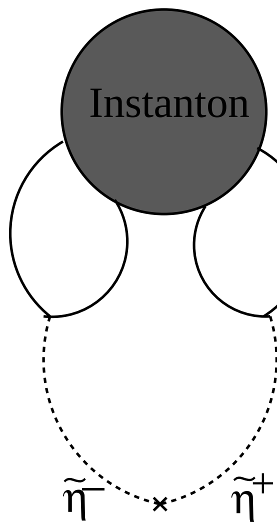

Figure 1. The $\langle\tilde{\eta}\tilde{\eta}\rangle$ condensate

The condensate that we calculate is

$$\Gamma_0 = \epsilon_{AB} \left\langle \tilde{\eta}^+_A(0)\, \tilde{\eta}^-_B(0) \right\rangle \,. \tag{9}$$

The relevant diagram is shown in Fig. 1. In this diagram, each scalar line emanates from a source which is the product of two fermionic zero modes. The two contributions to each source, arising from picking different channels of each zero mode, are depicted in Fig. 2. The condensate is given by[2]

$$\begin{aligned} \Gamma_0 &= c\, \epsilon_{kl}\epsilon_{AB} \int \frac{d^4x_0 d\rho}{\rho^5}\, e^{-S_E(\rho)} \int d^4z\, I_{kC}(z-x_0)\, G_{CB}(z,0;x_0) \\ &\quad \times \int d^4z'\, I_{lD}(z'-x_0)\, G_{DA}(z',0;x_0) \end{aligned} \tag{10}$$

where

$$I_{kC} = ig\sqrt{2}\, T^c_{CB}\, \eta_{B\alpha}\, \epsilon_{\alpha\beta}\, (\lambda^c_\beta)_k + y\, \epsilon_{ij}\epsilon_{BC}\epsilon_{\alpha\beta}\, \xi^\dagger_{j\beta}\, (\psi^\dagger_{Bi\alpha})_k \tag{11}$$

An integration over the SU(2) collective coordinates, which yields a factor of the group's volume, has been absorbed into the dimensionful constant c.

We will now show that Γ_0 is non-zero. First, it is a matter of straightforward algebra to show that

$$I_{kA}(x-x_0) = -i\, \epsilon_{kA}\, s(r)\,, \tag{12}$$

[2] The euclidean partition function of supersymmetric theories is defined using a Majorana representation for the fermions [14]. Consistency of this representation requires one to choose all the radial functions in eqs. (7) and (8) to be real.

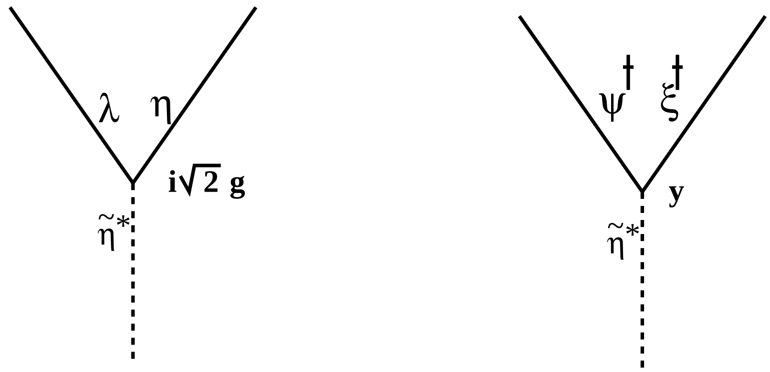

Figure 2. The two contributions to each source eq. (11)

where

$$s(r) = \frac{3g}{\sqrt{2}} u(r) f(r) + y(2g(r) + r^2 h(r)) v(r)\,. \tag{13}$$

In the large-r limit, $s(r)$ is dominated by the $h(r)v(r)$ term. (We assume $m < \mu$, see App. A). Hence, $s(r)$ cannot be zero everywhere. Its asymptotic behaviour is

$$s(r) \sim r^{-3} e^{-(m+m_1)r}\,. \tag{14}$$

Next, the scalar propagator is defined by the equation

$$\left(-D^2 + m_1^2 r^2 \varphi^2(r)\right) G(x, y; x_0) = \delta^4(x-y)\,. \tag{15}$$

It follows from this equation that

$$\epsilon_{AB}\epsilon_{CD}\, G_{BD} = G^*_{AC}\,. \tag{16}$$

Finally, we let

$$F(x, x_0) = \int d^4z\, G(z, x; x_0)\, s\left(|z - x_0|\right)\,. \tag{17}$$

Notice that $F(x, x_0) = F(x - x_0)$. Putting everything together, the condensate is

$$\Gamma_0 = c \int \frac{d^4x_0 d\rho}{\rho^5}\, e^{-S_E(\rho)}\, \mathrm{tr}\, F(x_0) F^\dagger(x_0)\,, \tag{18}$$

Eq. (14) and (15) imply that $F(x_0)$ cannot be zero everywhere, and that its asymptotic behaviour is the same as the η-propagator, i.e.

$$F(x - x_0) \sim r^{-\frac{3}{2}} e^{-m_1 r}\,. \tag{19}$$

This complete the proof that Γ_0 is non-zero.

If one considers an antiinstanton instead of an instanton, one finds a condensate of the complex conjugate fields, which satisfies the on-shell relation $\langle\tilde{\eta}^*\tilde{\eta}^*\rangle = \langle\tilde{\eta}\tilde{\eta}\rangle^*$.

Hence, the one-instanton result eq. (18) is actually the value of the condensate in the presence of a dilute instanton-antiinstanton gas.

4. Explicit SUSY breaking

The condensate discussed in the last section can be regarded as the point-like limit of the two-point function $\epsilon_{AB}\left\langle \eta^+_A(x)\eta^-_B(y)\right\rangle$. But this two-point function is not gauge invariant. Instead, we consider the gauge invariant two-point function related by complementarity

$$\Gamma(x,y) = \epsilon_{AB}\epsilon_{CD}\epsilon_{ij}\left\langle \tilde{\eta}^+_A(x)\phi_{iB}(x)\phi_{jD}(y)\tilde{\eta}^-_C(y)\right\rangle . \tag{20}$$

The leading order contribution to $\Gamma(x,y)$ is obtained by substituting the classical Higgs field of eq. (5) for $\phi_{iA}(x)$. In the point-like limit one obtains a new condensate $\Gamma(x,x) = \Gamma(0,0)$. Computing this condensate amount to almost exactly repeating the previous calculation. The result is

$$\Gamma(0,0) = c\, v^2 \int \frac{d^4x_0 d\rho}{\rho^5}\, e^{-S_E(\rho)}\, x_0^2\, \varphi^2(|x_0|)\, \mathrm{tr}\, F(x_0)F^\dagger(x_0)\,. \tag{21}$$

Hence, this condensate too is non-zero.

Now, the fields $\phi_{iA}(x)$ and $\eta^\pm(x)$ are all lowest components of chiral superfields. Unbroken SUSY requires the correlation function of any product of these fields (but not their complex conjugates) to be a constant, independent of the separation between points [5, 6]. We have seen above that $\Gamma(x,x)$ is non-zero. We will now prove that SUSY is explicitly broken in the one instanton sector, by showing that $\Gamma(x,y)$ *depends* on $x-y$. In fact, $\Gamma(x,y)\to 0$ as $|x-y|\to\infty$.

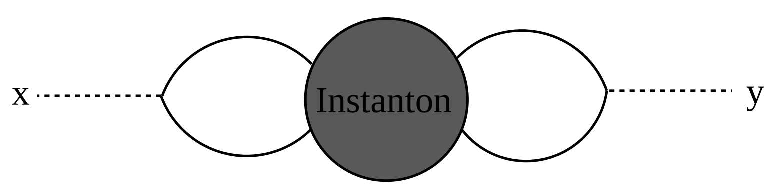

Figure 3. The two point function eq. (20)

$\Gamma(x,y)$ is given by the diagram shown in Fig. 3. Explicitly we find

$$\begin{aligned}\Gamma(x,y) \;&=\; c\int \frac{d^4x_0 d\rho}{\rho^5}\, e^{-S_E(\rho)} \\ &\quad \times \mathrm{tr}\, F(x-x_0)\,\phi^\dagger(x-x_0)\,\phi(y-x_0)\,F^\dagger(y-x_0)\,. \end{aligned} \tag{22}$$

Using eqs. (5) and (19) the asymptotic behaviour is

$$\Gamma(x,y) \sim |x-y|^{-\frac{1}{2}} e^{-m_1|x-y|}\,. \tag{23}$$

Thus, $\Gamma(x,y)$ tends to zero exponentially at large separations.

5. Inducing the Electro-Weak scale

In this section we illustrate a new mechanism for inducing the Electro-Weak scale via non-perturbative SUSY breaking effects. To this end, consider the following superpotential

$$W' = h' H^0 \left(\frac{1}{2} \epsilon_{i'j'} \epsilon_{A'B'} H_{i'A'} H_{j'B'} - \epsilon_{AB} \eta^+_A \eta^-_B \right)\,. \tag{24}$$

The $H_{i'A'}$ are now assumed to be the two Higgs superfields of the minimal supersymmetric Standard Model, and the primed indexes refer to $\mathrm{SU}(2)_L$. The model introduced in Sect. 2 is now regarded as a prototype for some higher scale physics, i.e. as a hidden sector.

The scalar potential that corresponds to eq. (24) contains a term

$$V = h'^2 \left| \frac{1}{2} \epsilon_{i'j'} \epsilon_{A'B'} \tilde{H}_{i'A'} \tilde{H}_{j'B'} - \epsilon_{AB} \tilde{\eta}^+_A \tilde{\eta}^-_B \right|^2\,. \tag{25}$$

Now, once the condensate $\langle \tilde{\eta}\tilde{\eta} \rangle$ is formed, this becomes (see eq. (9))

$$V = h'^2 \left| \frac{1}{2} \epsilon_{i'j'} \epsilon_{A'B'} \tilde{H}_{i'A'} \tilde{H}_{j'B'} - \Gamma_0 \right|^2\,. \tag{26}$$

As a result, the Higgs fields of the supersymmetric Standard Model develop an expectation value $\left\langle \tilde{H}_{i'A'} \right\rangle = \Gamma_0^{\frac{1}{2}} \delta_{i'A'}$, which breaks $\mathrm{SU}(2)\times\mathrm{U}(1)_Y$ to $\mathrm{U}(1)_{EM}$. Notice that going from eq. (25) to eq. (26) leaves the lagrangian of the observed sector supersymmetric. We do expect, however, that the inclusion of other effective interactions induced by the hidden sector will give rise to explicit SUSY breaking terms in the lagrangian of the observed sector.

Can this mechanism yield a phenomenologically acceptable value for the Electro-Weak scale? There are two possible sources for Γ_0. One scenario is that the condensate arises from GUT scale physics. In this case, the model of Sect. 2 should be regarded as a toy model for the relevant GUT scale physics.

The other scenario is based on the observation that the non-perturbative effects violate the SUSY algebra. Consequently, *negative* values for the vacuum energy are not impossible. One should investigate the possibility that, as in ordinary QCD, the non-perturbative effects in supersymmetric QCD *lower* the vacuum energy [15]. If

this is true, supersymmetric QCD will exist in a confining phase where the SUSY violating effects are $O(1)$. The Electro-Weak scale can then be induced by a strongly interacting hidden sector at the TeV range.

6. Discussion

In this paper we showed that one instanton effects in SUSY-Higgs models violate SUSY explicitly. How does this result compare with previous calculations? Non-perturbative SUSY breaking effects have already been found in ref. [9, 10]. We should mention in particular the demonstration that the S-matrix for elementary particle – soliton scattering is not supersymmetric already at tree level [10].

In the literature on supersymmetric Yang-Mills (SYM) there are one instanton calculations that give rise to supersymmetric results (see e.g. ref. [4, 5]). But SYM is a strongly interacting theory, and so the one instanton result in SYM is not a leading order contribution in any systematic expansion. For example, in the case of an SU(2) theory, the correlator $\langle \lambda\lambda(x)\lambda\lambda(y)\rangle$ is non-zero on the one hand, and it is required to be independent of the separation $x - y$ on the other hand [5, 7]. The one instanton contribution to this correlator is dominated by instantons whose size ρ satisfies $\rho \sim |x - y|$. Hence, the supersymmetric result is unreliable for separations which are large compared to the confinement scale. A similar statement applies to instanton calculations in supersymmetric QCD. Since the squarks' VEV can potentially be zero, one cannot rule out the possibility that the theory is strongly interacting *and* breaks SUSY explicitly at the same time.

The existence of explicit non-perturbative SUSY breaking effects raises some as yet unresolved issues. In the case of the chiral anomaly, the local continuity equation is violated in perturbation theory. This entails a violation of the axial charge at the non-perturbative level, whose manifestation is the occurrence of fermionic zero modes [12]. Since we have found that conservation of the SUSY charge is violated by non-perturbative effects, the question arises whether there is some indication from perturbation theory that this is going to happen.

Present day understanding of the perturbative properties of the SUSY current leaves open certain subtleties. At the moment, we would like to draw attention to some general differences between axial symmetries and SUSY. The chiral anomaly is a phenomenon that occurs at the level of a free fermion field in an external gauge field. In this setting perturbation theory has a finite radius of convergence. In fact, a fermionic determinant whose gauge variation is given exactly by the usual anomaly can be defined for non-perturbative gauge fields as well [16].

In the SUSY case, on the other hand, it is impossible to consider the gauge field as external without breaking the supermultiplet structure. Because of the non-linearity of the SUSY current, the definition of a conserved current can only be done order by order in perturbation theory [17]. In a full-fledged field theory, however, perturbation theory is only an asymptotic expansion, whose minimal error is given by the magnitude of non-perturbative effects. Thus, the perturbative construction only implies that violations of the conservation equation, if they exist, must be of a non-perturbative nature.

Appendix A. Asymptotic behaviour of the zero modes

In this appendix we discuss the asymptotic behaviour of the zero modes. The "lepton" zero modes solve the following set of ordinary differential equations

$$\begin{aligned} 2u' + 3a\,u + m_1\varphi\, v &= 0\,, \\ 2v' + m_1\varphi\, u &= 0\,. \end{aligned} \tag{27}$$

Here $a = a(r)$ and $\varphi = \varphi(r)$, see eqs. (4) and (5). The prime denotes differentiation with respect to r^2. The asymptotic large-r behaviour inferred from these equations is

$$u(r)\,, v(r) \sim r^{-\frac{3}{2}} e^{-m_1 r}\,. \tag{28}$$

In order to write down the equations for the gaugino zero modes we introduce the linear combination

$$h_1(r) = g(r) + 2r^2 h(r)\,. \tag{29}$$

The radial equations are

$$\begin{aligned} 2f' + 4af + (\mu/\sqrt{2})\varphi\, g &= 0\,, \\ 2g' + (r^{-2} - 2a)g + (a - r^{-2})h_1 + \sqrt{2}\mu\varphi f &= 0\,, \\ 2p' - (m/\sqrt{2})\varphi\, h_1 &= 0\,, \\ 2h_1' + (3/r^2)h_1 + 3(a - r^{-2})g - \sqrt{2}m\varphi\, p &= 0\,. \end{aligned} \tag{30}$$

The pairs (f, g) and (p, h_1) diagonalize the mass operator at infinity. Notice also that the mixed terms in the g- and h_1-equations are proportional to $a(r) - r^{-2}$, which decreases exponentially for large r. It will be convenient for us to consider the case $m < \mu$. The asymptotic behaviour of each channel is then determined by its own mass, and we find

$$h_1(r)\,, p(r) \;\sim\; r^{-\frac{3}{2}} e^{-mr}\,, \tag{31}$$

$$f(r)\,, g(r) \;\sim\; r^{-\frac{3}{2}} e^{-\mu r}\,. \tag{32}$$

Appendix B. Generalization to $N_f > 2$

In this appendix we show that the same pattern found for $N_f = 2$ generalizes to $N_f > 2$. Again, we will show that a correlation function which is required by SUSY to be a constant, fails in fact to be so.

The model with $N_f > 2$ is constructed as follows. Instead of two lepton families we now take $2M$ families where $M = N_f - 1$. The corresponding superfields are denoted $\eta_A^{n\pm}$ and $\xi_i^{n\pm}$, where $n = 1, \ldots, M$. We also introduce two new neutral superfields ω_1 and ω_2. These will form a massive Dirac fermion and two massive scalars which are singlets under all the internal symmetries except the non-anomalous R-symmetry. The role of the new scalars is to absorb the additional zero modes present for $N_f > 2$.

The superpotential is

$$W = W_1 + \sum_{n=1}^{M} W_2(\eta_A^{n\pm}, \xi_i^{n\pm}) + W_3 \,, \tag{33}$$

where W_1 and W_2 are given by eqs. (1) and (3) respectively, and

$$W_3 = m'\omega_1\omega_2 \,+\, y'\epsilon_{ij}\,\omega_1 \sum_{n=1}^{M} \xi_i^{n+}\xi_j^{n-} \,. \tag{34}$$

The general model, too, has a unique supersymmetric minimum, and the VEV-s of all the new scalar fields are zero. As in the $N_f = 2$ case, all fields acquire non-zero masses at tree level.

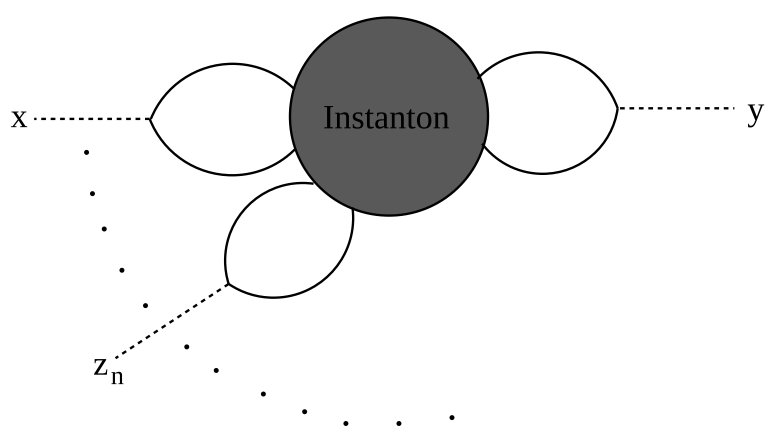

Figure 4. The correlation function eq. (35)

The gauge invariant correlator that unbroken SUSY would require to be a constant is (compare eq. (20))

$$\begin{aligned}\Gamma(x,y,z_1,\ldots,z_{M-1}) &= \epsilon_{AB}\epsilon_{CD}\epsilon_{ij} \\ &\quad\times\left\langle \tilde{\eta}^+_A(x)\phi_{iB}(x)\phi_{jD}(y)\tilde{\eta}^-_C(y)\tilde{\omega}_1(z_1)\cdots\tilde{\omega}_1(z_{M-1})\right\rangle . \quad (35)\end{aligned}$$

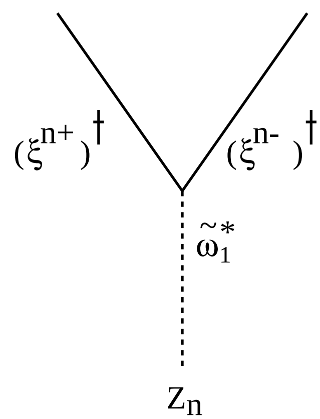

Figure 5. The source eq. (37)

One finds (see figs. 4 and 5)

$$\begin{aligned}\Gamma(x,y,z_1,\ldots,z_{M-1}) &= cM!\int\frac{d^4x_0d\rho}{\rho^5}\,e^{-S_E(\rho)} \\ &\quad\times\mathrm{tr}\,F(x-x_0)\,\phi^\dagger(x-x_0)\,\phi(y-x_0)\,F^\dagger(y-x_0) \\ &\quad\times T(z_1-x_0)\cdots T(z_{M-1}-x_0)\,, \quad (36)\end{aligned}$$

where

$$T(z-x_0) = 2y'\int d^4x\,v^2(|x-x_0|)\,G_1(x,z;x_0)\,, \quad (37)$$

and G_1 is the $\tilde{\omega}$-propagator. As in Sect. 4, one can show that in the point-like limit $x = y = z_1 = \ldots = z_{M-1}$ one obtains a non-zero condensate, whereas $\Gamma(x,y,z_1,\ldots,z_{M-1})$ tends to zero if the separation between any two points tends to infinity.

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superfield	fermion	boson	Q_R
V^a	λ^a	A_μ^a	1
Φ_{iA}	ψ_{iA}	ϕ_{iA}	-1
Φ^0	ψ^0	ϕ^0	1
$\eta_A^\pm$	$\eta_A^\pm$	$\tilde{\eta}_A^\pm$	-1
$\xi_i^\pm$	$\xi_i^\pm$	$\tilde{\xi}_i^\pm$	1

Table 1: The field content of the model of Sect. 2. The last row gives the fermion's charge under the non-anomalous R-symmetry.

channel	S_1	S_2	T	F	L	K_1	K_2
λ_α^a	$\frac{1}{2}$	0	1	0	0	$\frac{1}{2}$	0
$\psi_{iA\alpha}^\dagger$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0,1	$\frac{1}{2}$	0
ψ_α^0	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	0
$\eta_{\alpha A}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0	0
$\xi_{\alpha i}^\dagger$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0

Table 2: The channels of the fermionic zero modes and their quantum numbers.